Theories and Evidence of the Capital Market :

A Case Study in Hong Kong

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Abstract

The total risk of a particular capital asset can be decomposed into two components : the systematic risk and unsystematic risk. The unsystematic risks are those risks which are specific in determining the return of the asset, such as goodwill, management skills etc. The systematic risks represent those risks generated by the underlying economic forces which influence the returns of all the capital assets at the same time. Given a well diversified portfolio, we can diversify all the unsystematic risks. Therefore, there are only systematic risks that the market has to compensate the investors for bearing them. Traditionally, the Capital Asset Pricing Model (CAPM) is used to measure the risks of different assets. There have been several works on the risk and return on the stocks in Hong Kong by adopting the CAPM model. This thesis is a preliminary work using an alternative model, the Arbitrage Pricing Theorem (APT), to measure the systematic risk. Several empirical tests have been performed in order to compare the APT and the CAPM. Our data set

contains the daily closing prices of 32 Hang Seng Index constituent stocks and the Hang Seng Index itself from 1/12/1987 to 31/3/1990. The results are quite encouraging. The APT is better than the CAPM in measuring the system risk. Chapter one and Two are used to present the CAPM and the APT respectively. Chapter Three states the empirical models. Four tests have been conducted in this thesis. Test One and Two concern the APT itself. Test Three is used to compare the model specification of the APT in the presence of the CAPM. Test Four is a direct test between the APT and the CAPM. The result of the empirical work are reported in Chapter Four. We found that the APT is valid either in the presence of no alternative hypothesis or in the presence of alternative hypothesis, in our case, the "own" variance effect. But then in Test Three, we found that both the APT and the CAPM have model specification errors in the presence of each other. However, in Test Four, the results show us that the APT is much better in capturing the systematic risk than the CAPM.

CONTENTS :

CHAPTER 0 INTRODUCTION

Modern finance theory shows that the total risk of an asset can be decomposed into two parts : systematic risk and unsystematic risk. The unsystematic risks are those risks which are unique to a particular asset, such as goodwill, management skills etc.. The systematic risks, on the other hand, represent those risks generated by the underlying economic as well as political environment which influence the returns of all the capital assets at the same time. Given a well diversified portfolio, we can diversify all the unsystematic risks. Therefore, there are only systematic risks that the market has to compensate the investors for bearing them.

Traditionally, the Capital Asset Pricing Model(CAPM) is used to measure the systematic risks of different assets. There also have been several works on the risk and return on the stocks in Hong Kong by adopting the CAPM model. However, even though the CAPM has been predominant in empirical work, accumulating research has increasingly cast doubt on its ability to explain the cross-sectional difference in asset returns. The Arbitrage

Pricing Theory (APT), originally formulated by Ross and extended by Huberman, is an asset pricing model that offers a testable alternative to the well-known CAPM in explaining the cross-sectional variation in asset returns. The APT begins with an assumption on the return generating process. If the asset return is linearly related to the common factors and its own idiosyncratic components, then in a well-diversified, frictionless, and perfectly competitive economy the no arbitrage condition requires that the expected return be linearly related to the factor loadings which represent the influence of the common factors. In contrast to the CAPM, empirical evidence on the APT has been rather encouraging (see Roll and Ross(1980), Chen(1983), Dhrymes, Friend and Gultekin(1984), Lehmann and Modest(1988)). On the contrary, it is hard for us to find a similar work using the APT in studying the Hong Kong stock market except for S.N.Lee (1989). However, Lee's work was focuses only on the Hotel sector and explicitly assumes the validity of the APT as well as its advantages over the CAPM. He also specifies four common factors in addition to the market factor. But then, it is much better for us to test the APT and compare its performance with the CAPM by using the Hong Kong stock data rather than just assuming that it is true in our market.

This thesis is a preliminary work in this direction. Several empirical tests have been performed in order to compare the APT and the CAPM. Our data set contains the daily closing prices of 32 Hang Seng Index constituent stocks and the Hang Seng Index itself from 1/12/1987 to 31/3/1990. All prices are adjusted to dividends and stock splits.

In this paper, we will first derive the CAPM in chapter one and the APT in chapter two. We can see from these two chapters that the APT and the CAPM are non-nested since they are the results from different assumptions. We will set up the empirical model in chapter three. In test 1, we perform the basic test. It concludes that one factor is definitely present in the expected returns of equities traded on the Hong Kong Stock Exchange during the period from 1/12/1987 to 31/3/1990. Test 2 is much more important and powerful than test 1 , where the APT is compared against a specific alternative hypothesis that "own" variance influences expected returns. If the APT is true, the "own" variance should not be important, even though its sample value is known to be highly correlated cross-sectionally with sample mean returns. We find that after accounting for the factor loadings, the "own" variance does not have any significant influence on the expected return. In our work, we also try to compare the empirical performance of the APT with that of the CAPM. In test 3, we try to see whether the APT is specified more correctly than the CAPM. In the presence of the CAPM, the APT seems to be specified better than the CAPM. However, if we change the role of the APT and the CAPM in our analysis, we also find that the CAPM is specified more correctly than the APT. Therefore, by the model specification test, we cannot judge whether the APT is specified more correctly than the CAPM or not.In test 4, we directly compare the APT with the CAPM. If the APT is better than the CAPM in measuring risk, then the residuals generated from the APT should not be explained by the CAPM. On the other hand, if the CAPM is better than the APT, the residuals of the CAPM should not be explained by the APT. Our results show that the factor loading can explain the residuals of the CAPM significantly, while the converse is not true. Therefore the APT is better than the CAPM in capturing systematic risk. We summarize our findings in chapter four. The conclusion is in chapter five.

Chapter One The CAPM and Its Deficiency

The central idea of the capital asset pricing model (CAPM) is to determine the prices of capital assets under uncertainty in a competitive equilibrium framework. In a competitive equilibrium, a set of prices is determined such that the aggregate supply of capital assets equals aggregate demand. All investors will be satisfied with their optimal portfolio positions in equilibrium and there are no forces in the economy for any change. Given the basic assumptions of the CAPM which will be listed, the model states that in equilibrium, the expected return , E(Rj), on any asset j will be given by the equation(1) :

$$E(R(X_j)) = R_F + \lambda [cov(R(X_j),R(m))/ \sigma(R(m)]$$

where R_F is the risk-free rate of return. And $\lambda = [E(R(m)) - R_F]/ \sigma(R(m))$ is the market risk premium per unit of risk. E(R(m)) is the expected return on the market portfolio which is defined to be any portfolio such that the weight, MKT_j, of this portfolio equals the total market value of assets j's shares to the total market value of all the risky assets shares outstanding. $\sigma(R(m))$ is the standard deviation of return on the market portfolio, and $Cov(R(X_j), R(m))$ is the covariance between the return on asset j and the return on the market portfolio. We now summarize the major assumptions of the CAPM and try to interpret the economic rationale behind that model.

The model has two dates times 0 and 1. At time 1, the uncertainty in the economy is characterized by a state space Ω. A capital asset X is defined as it's dollar cash flows across all states $W \in \Omega$ at time 1, i.e. $X(W) : \Omega \longrightarrow \mathbb{R}$. The set of assets that trade at time 0 is denoted by M. The price of any asset at Time 0 is $P(X_p) \neq 0$ therefore, the return on any asset $X \varepsilon M$ is well defined by $R(X) = X-P(X)/P(X)$. And there is no inflation.

A.1 If assets x, y ε M, then ax + by εM for all a, b $\varepsilon\mathbb{R}$. i.e. the marketed assets are closed under the construction of portfolio.

A.2 All investors are single-period expected utility of return maximizers. Their utility function U(R) is either in quadratic form or the returns follow a two-parameter probability density function $f(R_j; E(R), Var(R))$. Therefore, the expected utility will only depends on the expected return $E(R)$ and the variance of R, $Var(R)$. The investors will choose among alternative portfolios on the basis of $E(R)$ and $Var(R)$ [2].

A.3 For all investors, their utility function satisfies the monotonicity assumption and has continuous second-order partial derivatives. Also the differentiation (up to the second order) under the expectation is a valid operation.

A.4 All investors are risk-averse and are price takers.

A.5 All investors can borrow or lend an unlimited amount at an exogenously given risk-free rate of interest R_F.

A.6 All investors have identical subjective estimates of the means, variances and covariances of return among all assets. Besides, the $E(X^2)$ is assumed to be less than infinitive, i.e. $E(X^2) < +\infty$ in order to guarantee the existence of means, variances and covariance [3].

A.7 All assets are perfectly divisible and perfectly liquid, i.e. all assets are marketable and there are no transactions costs, no taxes and no restrictions on short sales of any asset.

A.8 The quantities of all assets are given and are strictly greater than zero, i.e. there are positive supply for any asset.

By A.2, A.3 and A.4, the indifference curves on the mean-variance plane all have positive slopes and are concave upwards [4].

In Fig 1, the shaded area represents all possible combinations of risk and return available from investments in the risky assets for the investor.

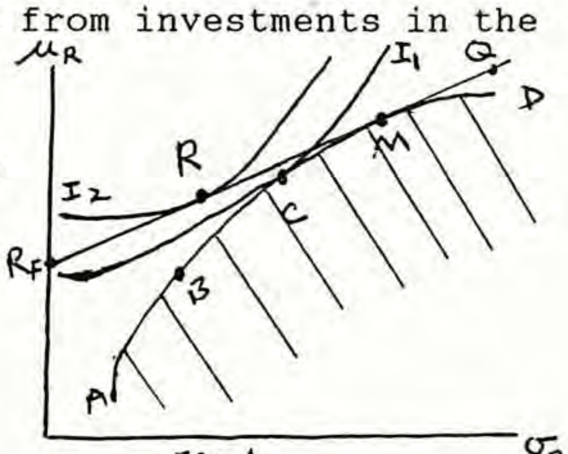

Fig 1

The portfolios lying on the boundary ABMD represent the set of mean-variance efficient portfolios [5]. A portfolio $(W_1, ..., W_K)$ of risky assets with an expected return of U is said to be mean-variance efficient if $Var(\sum_{j-1}^{K} W_j R(X_j)) \le Var(\sum_{j-1}^{K} N_j R(X_j))$

satisfies for all portfolios $(N_1, ..., N_K)$ such that $\overline{U} - E(\sum_{j-1}^{K} W_j R(X_j)) - E(\sum_{j-1}^{K} N_j R(x_j))$

In other words, for a given expected return $\overline{U}$, the mean-variance efficient portfolios minimizes the variance of the portfolio's return over all portfolios having the same expected return. If we let $\overline{U}$ vary, we will get the set of the mean-variance efficient portfolios. Although we will not show that the set of points can be represented by a hyperbola as graphed in Fig 1, we can find the proof in a paper written by Merton(1972). The investor will only choose the efficient portfolios lying on the boundary BCMD since these portfolios give him the maximum expected return at a given variance(6). In the absence of risk-free asset, the optimal solution to the portfolio problem for the investor is point C, which represents the portfolio, where the indifference curve I_1 is tangent to the boundary BCMD. If there exists a risk-free asset, the investor will select among the portfolios which are the linear combinations of the risk-free asset and the risky asset portfolio M . We can see that all the mean-variance efficient portfolios will be dominated by the portfolios lying on the lineR_FMSsince these portfolios are having a higher or equal expected return for a given variance than the portfolios on the boundary BCMD. Therefore, in order to maximize the expected utility, the investors are inclined to choose the portfolio R and enjoy a higher expected utility represented by I_2.

Before we go further, we should note that the risky assets portfolio must be the market portfolio otherwise the capital asset market will not be in equilibrium. A market portfolio is a portfolio which consists of every assets and the market portfolio weight MKT_j equals the total market value of asset j's shares to the total market value of all the risky asset shares, i.e. $MKT_j=\sum_{i-1}^{N} \bar{N}_j^i P(X_j)/\sum_{k-1}^{K} \sum_{i-1}^{n} N_K^i P(X_K)$ In general, if the optimal portfolio M does not contain every asset, then the prices of those assets which are not in the portfolio will go down. Therefore, they will become more attractive and the investors will be willing to include them in their portfolios. As a result, capital asset prices will continue to change until a set of prices is attained for which every asset enters the optimal portfolio, i.e. the market portfolio.(7).

Most important, however, is the result that the expected return of any portfolio lying on the line, for example Q, can be expressed as a linear relationship with the expected return of the market portfolio.

i.e. $E(R_Q) = R_F + [(E(R_m) - R_F) / \sigma(R_m)] \sigma(R_Q)$

Suppose the investor i buys N_j^i shares of the jth asset and j = 0 represent the riskless asset, then the total return in time 1 is $R=\sum_{j=0}^{K} N_j^i X_j$. The expected return flow is $E(R) = \sum_{j=0}^{K} N_j^i E(X_j)$ and the variance of the cash flow is $Var(R) = \sum_{j=0}^{K} \sum_{K=0}^{K} N_j^i N_K^i Cov(X_j, X_K)$. . As a result, the investor's portfolio problem can be formulated as choosing the optimal amount of $(N_0, \ldots, N_K)$ such that he can maximize his expected utility V(E(R), Var(R)). That is, Max. V(E(R), Var(R)) where the constraint is simply the budget constraint which states that the investor i can sell his endowed portfolio N_j^i in order to buy the desired portfolio N_j^i.

Let $$L=V(\sum_{j=0}^{K} N_j^i E(X_j), \sum_j \sum_K N_j^i N_K^i Cov(X_j, X_K)) + \lambda (\sum_j \overline{N_j^i} P(X_j) - \sum_j N_j^i P(X_j))$$

then

F.O.C.

$$/\partial N_j = (\partial V(.)/\partial ER)(E(X_j)) + (\partial V(.)/\partial VarR)(2\sum_K N_K Cov(X_j, X_K)) - \lambda P(X_j) = 0 \quad (1)$$

$$\partial L/\partial\lambda = \sum_j \overline{N}_j^i P(X_j) - \sum_j N_j^i P(X_j) = 0 \qquad (2)$$

For j=o, $E(X_O) = X_O$ and cov $(X_O, X_K)=0$

since j=o represents the risk-free asset. Therefore,

$\lambda = (\partial V(.)/\partial ER)(X_o/P(X_o)) = (\partial V(.)/\partial ER)(1+R_F)$ and (1)

can be rewritten as

$$\partial V(.)/\partial ER]\,E(X_j) + [\partial V(.)/\partial VarR](2\sum_K N_K^i Cov(X_j, X_K)) - [\partial v(.)/\partial ER]\,[X_0/P(X_0)] - P(X_j)$$

$$\Rightarrow [\ \partial V(.)/\partial ER\ ][\ E(X_j)/P(X_j)\ ] - [\ \partial V(.)/\partial ER\ ][\ X_o/P(X_o)\ ]$$
$$+\ 2\ (\partial V(.)/\partial VarR)\ \ [\sum_K N_K^i cov(X_j/P(X_j), X_K)]\ = 0$$

$$\Rightarrow [\ \partial V(.)/\partial ER\ ][\ E(X_j)/P(X_j)\ ] - [\partial V(.)/\partial ER](X_0/P(X_0))$$
$$+\ 2\ (\partial V(.)/\partial VarR)\ \ [\sum_K N_K^i cov(X_j/P(X_j), X_K/P(X_K)]\ = 0$$

$$\Rightarrow [\ \partial V(.)/\partial ER\ ][\ (E(R(X_j) - R_F)\ ]$$
$$+\ 2\ (\partial V(.)/\partial VarR)\ (\sum_K N_j^i P(X_K)\,Cov(R(X_j), R(X_K))) = 0$$

$$\Rightarrow E(R(X_j) - R_F) = A^i(\sum_K N_k^i\, P(X_K)\,Cov(R(X_j), R(X_K))) \qquad (3)$$

where $A^i \equiv -2[(\partial V(.)/\partial VarR)/\partial V(.)/\partial ER)]$

$$\Rightarrow E(R(X_j) - R_F) = A^i \sum_K N_K^i P(X_K) \sum_{k=1}^{K} W_K^i Cov(R(X_j), R(X_K) \qquad (4)$$

where $W_K^i \equiv N_K^i P(X_K) / \sum_{K-1}^{K} N_K^i P(X_k)$ and $\sum_{K-1}^{K} W_K^i - 1$

we will show that $\sum_{K-1}^{K} N_K^i P(x_K) \neq 0$ in the Appendix. Note that $\sum_{K-1}^{K} W_K^i Cov(R(X_j), R(X_K)) - Cov(\sum_{k-1}^{K} W_K^i R(X_j), R(X_K))$

$$-Cov(R(m), R(X_j))$$

where R(m) is the return on the optimal portfolio which is also the market portfolio. Therefore

$$\sum_{j-1}^{K} W_j^i E(R(X_j)) - R_F$$

$$-\sum_{j-1}^{K} W_j^i Cov(R(m), R(X_j)) A^i (\sum_{k-1}^{K} N_K^i P(X_K))$$

Note that , $\sum_{j-1}^{K} W_j^i E(R(X_j)) - E(R(m))$ *and*

$$\sum_{j-1}^{K} w_j^i Cov(R(m), R(X_j)) - Cov(R(m), R(m)) - Var(R(m))$$

As a result, $E(R(m)) - R_F - A^i (\sum_{k-1}^{K} N_k^i P(X_k)) Var(R(m))$

and $A^i \sum_{k-1}^{K} N_k^i P(X_k) - E(R(m)) - R_F / Var(Rm)$ *substitute into* (4)

We get $E(R(X_j)) - R_F + (E(R(m)) - R_F / Var(R(m)) \; Cov(R(m), R(X_j))$

$$-R_F + \lambda Cov(R(m), R(X_j)) \forall j-1, \ldots, K.$$

Although this result still depends on i, however, we will show in the Appendix that this formula is in fact independent of i. Therefore, we have proved the fundamental result of the CAPM which states that the equilibrium expected return on any asset is equal to the riskless rate of return plus a risk premium which is the product of market risk premium and the risk of the j th asset as measured by cov(R(m), $(R(X_j))/(var(R(m))$. This result is appealing since we know that through diversification, some of the risks specific to the asset can be avoided, therefore, the total variance or the total risk of this asset is not the relevant measure of its risk. It is that part of an asset's risk which is due to its correlation with the return on the market portfolio which cannot be diversified[8]. Since the market portfolio includes all the assets, we can percieve that the return of the market portfolio is influenced by common economic activities. Since we can not diversify the risk of the economy as a whole, investors have to pay a premium in order to avoid this inescapable risk which is represented by the covariance of an asset with the market portfolio.

Even though the CAPM provides us with a better concept and tool in measuring the risk of different assets, this model is still subject to criticisms on its testability.[9] The central difficulty in testing this model comes from the fact that the efficient market portfolio and the CAPM are joint hypothesis which are not separable. This is because if the market portfolio is not mean-variance efficient, then the equilibrium condition of the capital assets will not necessarily hold. Therefore, if we really want to know the validity of the CAPM, we have to know whether the true market portfolio is mean-variance efficient or not. However, not all the traded assets have easily accessible or published price data. Also the market portfolio contains all the assets including marketable and nonmarketable ones, thus making it impossible to observe the return on this portfolio(10). Some economists try to test the CAPM by employing a proxy for the market portfolio and test the model with respect to a subset of all the assets. But this kind of test can only verify whether the market proxy is efficient or not. The results of these tests tell us nothing about the true market portfolios(11). Therefore, we still leave the problem unsolved. As a result, the test of the CAPM will be very inconclusive, because we may reject the CAPM while it is true or we may accept it while it is wrong. Despite the problem of testability, the CAPM still gains a lot of the attention from the public. The major reasons behind this phenomenon are that : firstly, the CAPM gives us a simple nice interpretation in the measurement of risk. Secondly, also the most important, is that the CAPM gives us a simple linear model of the risk of economy. It is easy to employ it to form corporate policies and do empirical research on this model. Therefore, it will be interesting if we can find a model which retains the simple linear feature and at the same time offers us a testable alternative to the capital asset pricing model. The arbitrage pricing theory (APT) which will be discussed in the next chapter gives us the answer.

APPENDIX 1

Proposition 1 :In equilibrium, the investors will choose an optimal portfolio while contains all the assets[14].

Proof :

since $A^i=-2(\partial V(E(R),Var(R)/\partial Var(R))/(\partial V(E(R),Var(R))/\partial E(R))>0$

from 3. $E(R(X_j)-R_f)=A^i\sum_i N_K^i P(X_k)Cov(R(X_j,R(X_k))\forall j=1....K$

It can be rewrited as R = CB where R is the matrix of $E(R(X_j)-R_F)$. C is the covariance matrix and B is the matrix of $A^iN_K{}^iP(X_K)$ since X1,... X_K are linearly independent therefore C is nonsingular. Hence C^{-1} R=B.

Let e_{ij} to be the(i,j)th elementof C^{-1}, then we get

$\sum_{j=1}^{K} e_{KJ}[(E(R(X_j))-R_F)= A^iN_K^ip(X_K)$

since the left hand side of this equation depends only on the probability belief which is assumed to be homogeneous. Among all the investors, therefore, the sign of $A^iN_K{}^i$ $P(X_K)$ is the same as the sign of $A^jN_K{}^j$ $P(X_K)$ for all i and j. Together with the fact that A^i, A^j, $P(X_K)$ are all greater than zero. We know that if $N_R{}^i > 0$ then $N_K{}^j >0$ which is true for all K=1, ..., K. Since at equilibrium $0<\sum_{i=1}^{n} N_K^i-\sum_{i=1}^{n} N_K^i$, Therefore, investor i should hold positive amount of asset K, i.e. $N_K^i > 0$. This implies that all other investors hold a positive amount of asset K and this is true for all assets.

Proposition 2. Since W_K^i is independent of i, therefore the result of the CAPM is also independent of i[15]. Since

$$\sum_{j=1}^{K} e_{KJ}[E(R(X_j)-R_F)] = A^i N_K^i p(X_K)$$

$$A^i = [E(R(m))-R_F]/[Var(R(m))\sum_K N_K^i p(X_K)$$

$$so that \sum_j e_{ij}[E(R(X_j)-R_F) = W_K^i E(R(X_j)-R_F)/Var(Rm) \quad \forall j=1,...,K$$

$$where\ W_K^i = (N_K^i p(X_K))/(\sum_K N_K^i p(X_K)) \quad and\ R(m) = \sum_j W_J^i R(X_j)$$

Therefore we get K equation and K unknowns W_K^i, Note that the solution, W_K^i will be independent of i.

Proposition 3, the weight W_K^i of the optimal portfolio is the same as the weight of the market portfolio MKT_K.

Since

$$MKT_K - \sum_i \bar{N}_j^i p(X_j) / \sum_K \sum_i \bar{N}_K^i p(X_K) - \sum_i N_j^i p(X_j) / \sum_K \sum_i N_K^i p(X_K)$$

the result of the market equilibrium.

$$\mathrm{MKT}_K = \sum_i N_j^i p(X_j) / \sum_i \left(\sum_K N_K^i p(X_K)\right) - \sum_i W_j^i \sum_j^K N_j^i p(X_j) / \sum_i \left(\sum_K N_K^i p(X_K)\right) - W_K^i$$

by proposition 1

As a result, by propositions 1, 2 and 3 we know that, in equilibrium, all investors will hold the market portfolio which is mean-variance efficient since investors will only hold the efficient protfolio.

Chapter One

Footnotes

(1) Jensen, Michael C., (1972) pp.359

(2) Tobin, James (1958)

(3) Jarrow, Robert A. (1908). pp.112

(4) Tobin, James (1958) pp.174-177

(5) Jarrow. Robert A. (1988) pp.226

(6) Jensoen, Michael c. (1972) pp.360

(7) Sharpe, William F. (1964) pp.225

(8) Copeland and Weston, Ch.7 pp.198-199

(9) See Roll (1977)

(10) Copeland and Weston, Ch.7 pp.217-219

(11) Jarrow. Robert A (1988) pp.233-234

(12) Jarrow. Robert A (1988) pp.199-201

(13) Tobin, James (1958) pp.174-177

(14) Jarrow, Robert A. (1988) pp. 214-220

(15) Ibid.

Chapter Two   The APT and Its Advantages Over The CAPM

The Arbitrage Pricing Theory(APT) rests on a simple fact that when the asset market is in equilibrium, there does not exist any arbitrage opportunity. We have to notice that no arbitrage opportunity is only the necessary condition for equilibrium to exist and the APT is simply derived from the no arbitrage situation. That is to say, the APT does not only hold for equilibrium situation, but it is also valid for disequilibrium situation. The basic idea behind the APT is that when an asset does not have any risk, then this asset should only earn the riskless rate of return, R_F. In other words, if the asset does not bear any risk and costs us nothing, then it will not have any return(1). Supposing that the returns of these assets are greater than zero, then every investor can get infinite money by holding them. No arbitrage opportunity tries to preclude this kind of situation. On the other hand, the total risk of a risky asset can be decomposed into two components, systematic risk and idiosyncratic risk(2). The APT asserts that systematic risk is caused by several common factors which represent the underlying economic forces that affect the return of every asset systematically. Idiosyncratic risk, on the contrary, only reflects the influence of the factors which are specific to a particular asset. Therefore, we can diversify the idiosyncratic risk by forming well-diversified portfolios. However, there is no way to diversify the systematic risk and it will be borne by the investors. As a result, the excess expected return, the expected return minus R_F, of any risky asset should only reflect the risk premium of those common factors. The market should compensate the investors in bearing the systematic risk. Before we go to the formal model of the APT, it is better to grasp some basic features of APT by studying a simple formulation of it(3). Suppose the return of asset i is generated by a linear stochastic process.

i.e. $R(X_i) = E(X_i) + B_iF + U_i \quad , \quad i=1,...,n$

in matrix notation, $R(X)_{nx1} = E(X)_{nx1} + B_{nx1}F_{1x1} + U_{nx1}$

The random return of asset i, $R(X_i)$, is linearly related to one common factor, F, and the specific factor, U_i. Now we form an arbitrage portfolio, N , which does not have any risk, and uses no wealth N'e = 0 where e is a vector matrix of 1. In other words, the wealth invested long in assets is exactly balanced by the amount borrowed from short sales. As a result, the return on this portfolio is given by $N'R(X) = N'E(X) + N'BF + N'U = N'E(X)$ We can always choose the arbitrage portfolio in such a way that it is well diversified and it eliminates all the systematic risk. Together with the fact that their portfolio does not use any wealth, it follows that its return, $N'E(X) = 0$. If this does not hold, we will be able to obtain arbitrarily large return by simple scaling up the arbitrage portfolio. This is incompatible with the absence of arbitrage, let alone equilibrium. Since N are orthogonal to e and B are orthogonal to E, therefore E must be a linear combination of e and B. Hence, there are constants E and a such that for all i,

$$E_i = E_0 + aB_i$$

Consider the following example. Suppose we have three assets which do not have any idiosyncratic risk and are jointly affected by one common factor. Their expected return and associated systematic risk are graphed in Fig 1.The APT simply says that when there is no arbitrage opportunity,these three points should lie on the line, $E_i = E_0 + aB_i$.

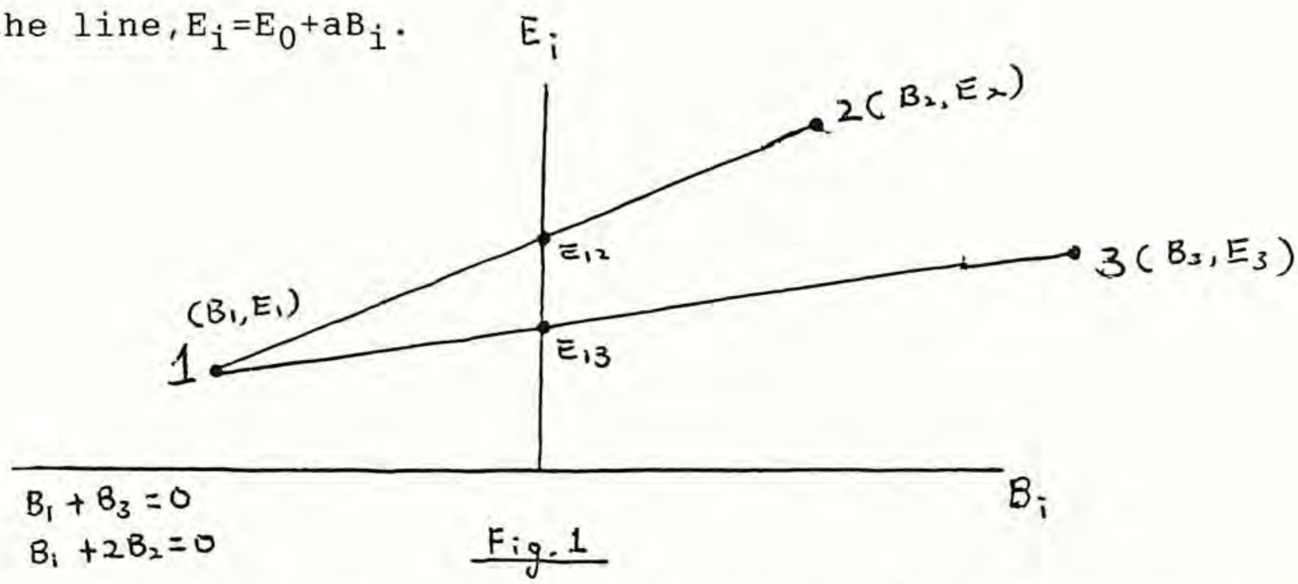

Fig. 1

In Fig 1, they do not lie on a single line. By investing the same amount of wealth in asset 1 and 3, we can form a portfolio earning the riskless rate of return, E13. However, By combining the first and second assets with investments of 1/3 and 2/3 of wealth respectively we obtain the riskless return E12 which is higher than E13. Therefore, If we sell short the E13 portfolio and go long in the E12 portfolio, we can receive arbitrarily high return without risk and cost. To prevent this we must have E12 = E13. In other words, the three points should lie on the same line. In concluding, the APT states that when there is no arbitrage opportunity and the asset return is following a linear generation process, then the expected return of this asset should be linearly related to the risk-free rate and the factor loading of the common factors. This result is not dependent on the equilibrium condition, therefore, the APT can hold under equilibrium as well as disequilibrium conditions.The basic setting of the model is quite the same as the CAPM in Chapter 1[4]. The model has two dates, time 0 and 1. At time 1, the uncertainty in the economy is characterized by a state space Ω . An asset X is defined as its dollar cash flows across all state $W \in \Omega$ at time 1, i.e. $X(W) : \Omega \longrightarrow \mathbb{R}$.

The set of assets that trade at time 0 is denoted by M . The price of any asset at time 0 is $P(X) \neq 0$, therefore, the return on any asset $X \in M$ is well defined by $R(X) = [X-P(X)] / P(X)$ and there is no inflation.

However, we can price some subset of M with the known prices of another subset of M . In fact, the asset in the algebraic basis will form the fundamental set of traded asset from which all other traded assets will be priced. The algebraic basis is the minimal set of asset $\{X_g\}_{g \in G}$. The X's in this set are linearly independent to each other. Therefore, given an asset X in M, it can be constructed in a unique way as a finite

asset portfolio of the minimal set $\{X_g\}_{g\epsilon G}$, i.e. $x = \sum_{g\epsilon g} N_g X_g$
If the price is following the value additivity which will be shown later as equivalent to no arbitrage opportunities, then

$$P(X) = \sum_{g\epsilon G} N_g P(X_g)$$

As a result, if we know the price of the basis assets, then we can price all other assets in M. Before we go to the details of the assumptions of the model, it is better for us to know precisely what is meant by arbitrage opportunities. There are two kinds of arbitrage opportunities. The finite and infinite asset arbitrage opportunities.(5) A finite asset arbitrage opportunity is a portfolio $\sum_{i=0}^{K} N_i X_i \; \epsilon \; M$ such that any one of the following four conditions is satisfied :

(1) $P(N_0 X_0 + \ldots + N_K X_K) \neq \sum_{j=0}^{n} N_j P(X_j)$

(2) For some i, Prob_i : ($\sum_{j=0}^{n} N_j X_j = 0$) = 1 and

(3) (a) $\sum_{j=0}^{n} N_j P(X_j) \leqslant 0$ and for some i

(b) Prob_i ($\sum_{j=0}^{n} N_j X_j \geqslant 0$) = 1 and

(c) Prob_i ($\sum_{j=0}^{n} N_j X_j > 0$) > 0

(4) (a) $\sum_{j=0}^{n} N_j P(X_j) \geqslant 0$ and for some i

(b) $\text{Prob}_i (\sum_{j=0}^{n} N_j X_j \leqslant 0) = 1$ and

(c) $\text{Prob}_i (\sum_{j=0}^{n} N_j X_j < 0) > 0$.

condition 1 states that an arbitrage opportunity is a portfolio whose price as a package differs from the prices of the assets purchased separately and then combined. Suppose

$$P(\sum_{j=0}^{n} N_j X_j) < \sum_{j=0}^{n} N_j P(X_j)$$

then we can make infinite profit by short selling N_j shares of X_j, for j=0, ... K, separately and by the package; $\sum_{j=0}^{n} N_j X_j$. Condition 2 says that the time 1 cash flow of the portfolio,$\sum_{j=0}^{K} N_j X_j$ is zero with probability 1. But the time 0 price of this portfolio is non zero. Suppose $P(\sum_{j=0}^{K} N_j X_j) > 0$ .A clever investor would short this package, generating $P(\sum_{j=0}^{K} N_j X_j)$ as a cash flow at time 0 but then he does not bear any liability in time 1. This is a money pump! We next consider (3) and (4). Since these conditions are similar, we only concentrate on condition (4). Condition (4a) state that the cash flow to purchasing N_j shares units of X_j for j=0, ... K at time 0 is zero or positive. By (4b), the portfolio will never have a positive cash flow in time 1 and with positive probability it has a non-positive cash flow at time 1(4c). This is a probabilistic money pump! It can be shown that by holding this portfolio, the investor can increase his utility at no risk and without cost[6]. In fact, arbitrage opportunity can be perceived as a utility-increasing trading strategy that do not bring any risk. On the other hand, The infinite asset arbitrage opportunities refers to the following situation :

For some investor s, $\sum_{j-0}^{n} W_j^n - 1$ for all n and $\lim_{n->\infty} Var_s(\sum_{j-0}^{n} W_j^n (X_j)) = 0$

and $\lim_{n->\infty} E_s (\sum_{j-0}^{n} W_j^n R(X_j)) = K > R_F$. This condition states that if for some investor, a sequence of portfolios exists with a positive investment; $\sum_{j-0}^{n} W_j^n - 1$ and a variance that goes to zero, but the expected return of the limiting portfolio is higher than the riskless rate, then a smart investor can have sure return, $K- R_F > 0$ without incurring any cost and risk. The investor can simply borrow fund at the risk-free rate and reinvest the fund into the above sequence of portfolios. The definition simply extends the finite arbitrage opportunity from finite asset portfolio to infinite asset portfolio. It is easy to show that the existence of infinite arbitrage opportunity gives a chance for the investor to increase his utility at no cost and no risk(7).

Since some of the basic assumptions in deriving the APT are the same as those used in the CAPM, therefore, we will only state them by their corresponding symbols. The other new assumptions will be discussed in details.

A.1 The marketed assets are closed under the construction of portfolio and it consists of all finite portfolios using the countable set of primary assets $\{X_j\}_{j=1}^{\infty}$ and X_Ω , the risk-free asset ϵ M . This assumption guarantee that there is at most a countably infinite number of primary assets. The idea of this assumption is to approximate the actual economy with an economy in which there are a countably infinite number of primary assets. We have to notice that M needs not include all assets. In fact, the traded assets studied, M , can even be a subset of all the "traded" assets.

A.2 The utility function satisfies the monotonicity assumption and has continuous second-order partial derivatives. Also the differentiation (up to the second order) under the expectation operator is a valid operation. The utility function only depends on the cash flow in time 1.

A.3 All investors are price takers.

A.4 All investors can borrow or lend an unlimited amount at an exogenously given risk-free rate of interest R_F.

A.5 Given an event H, $Prob_i(H)=0$ if and only if $Prob_j(H)=0$ for all investors i, j. This assumption states that investors agree on zero probability events. It implies that they need not agree on expected values or variance of the asset's cash flow.

A.6 For any traded asset $X \in M$, $E(X^2) < +\infty$ for all investors.

A.7 All assets are perfectly divisible and perfectly liquid.

A.8 No finite arbitrage opportunity

A.9 No infinite arbitrage opportunity.

A.10 There is no inflation in our model

A.11 Linear factor hypothesis

$R(X_j) = R_F + \sum_{k-1}^{n} \lambda_{jk} (f_K - R_F) + \lambda_{j,K+j} (U_j - R_F)$

$j = 1,2,...,$

and $Cov_s (U_i U_j) = 0$ if $i = j$, and for all investors s .

and Vars($\lambda_{j,K+j}$ (U_j - R_F)) < $\bar{\sigma}^2$ for all j and s where. $R(X_j)$ is the rate of return on the jth primary assets; R_F is the risk free rate of return. f_K is the return on the kth common factor. i.e. this return is common to all primary assets. U_j is the return specific to asset X_j. λ_{jk} is the coefficient. This assumption states that the return on any primary asset is generated from a linear process. This process contains the risk free rate, the common factors which affect all the assets systematically and the residual which represents the specific factors influencing a particular asset.

Given assumptions A1-A5 and A7-A10, we can demonstrate that the return on any primary asset can be expressed as a linear combination of the return on the basic assets.

Theorm 1 : $R(X_j) = \sum_{g \in G} g(X_j) R(X_g)$, j=1,2,... for a finite number of primary asset. Where $\lambda g(X_j) - N_g(X_j) P(x_g) / \sum_{g \in G} N_g P(X_g)$ and only a finite number of $\lambda_{.g} \neq 0$ and $\sum_{g \in G} \lambda g(X_j) - 1$.

Proof. Suppose that there is a portfolio, N, consisting of all the basis assets $\{ X_g \}_{g \in G}$, g=1,2,...,∞ , then the rate of return of this portfolio is

$$[\sum_{g\in G} N_g X_g - \sum_{g\in G} N_g P(X_g)] / \sum_{g\in G} N_g P(X_g)$$

$$P(\sum_{g\in G} N_g X_g) - \sum_{g\in G} N_g P(X_g) \text{ by A.8}$$

$P(X_g) > 0$ Therefore, the rate of return of N is well defined.

$$[\sum_{g\in G} N_g X_g - \sum_{g\in G} N_g P(X_g)] / \sum_{g\in G} N_g P(X_g)$$

$$= \sum_{g\in G} N_g (X_g - P(X_g)) / \sum_{g\in G} N_g P(X_g)$$

$$= \sum_{g\in G} [N_g P(X_g) / \sum_{g\in G} N_g P(X_g)] [(X_g - P(X_g)) / P(X_g)]$$

$$= \sum_{g\in G} \lambda_g R(X_g)$$

by A.1 Any primary asset can be written as a linear combination of the basis assets $X_i = \sum_{g\in G} N_g(X_i) X_g$ where there is a finite number of $N_g(X_i) \neq 0$

$$\therefore R(X_i) = \sum_{g\in G} \lambda_g R(X_g)$$

$$R(X_i) = R_F + \sum_{g\in G} \lambda_g (R(X_g) - R_F)$$

and $E_s(R(X_i)) = R_F + \sum_{g\in G} \lambda_g (E_s(R(X_g)) - R_F)$

where $\lambda_g(X_i) \neq 0$ for only a finite number of $g\in G$

As a result, The excess return on any primary asset can be expressed as a finite linear combination of the excess expected return on the basis assets. Now, we can split the basic assets into two sets. The return on the first k basis assets are (f_g) $g = 1, \ldots K$. and the return on the other set of basis assets is (U_j) $j = K+1, \ldots,$. Therefore

$$R(X_i) = R_F + \sum_{g=1}^{K} \lambda_{ig}(f_g - R_F) + \sum_{j=k+1}^{\infty} \theta_{ij}(U_j - R_F)$$

By A.11 there is only one coefficient of Uj which is not equal to zero and there is no restriction on the coefficient of the f_g. Therefore, A.11 just puts some restriction on Thm.1 . i.e.

$$R(X_i) = R_F + \sum_{g=1}^{k} \lambda_{ig}(f_g - R_F) + \theta_{ij}(U_j - R_F)$$

$$E_s(R(X_i)) = R_F + \sum_{g=1}^{k} \lambda_{ig}(E_s(f_g) - R_F) + \theta_{ij}(E_s(U_j) - R_F)$$

From the above assumptions, we can develop the APT which states that on the average, the expected return on any primary asset can be expressed as a linear combination of the excess return on the common factors.

$$E_s(R(X_i)) - R_F \approx \sum_{g=1}^{k} \lambda_{ig}(E_s(f_g)-R_F)$$

Proof : We first prove that $\lim_{n\to\infty} \sum_{j=1}^{n} \theta_{ij}(E_s(U_j)-R_F)^2 < +\infty$. Suppose not, then $\sum_{j=1}^{n} \theta_{ij}(E_s(U_j)-R_F)^2 = +\infty$ for some investor s . For convenience, let the return on X_j equal f_j for $j = 1, \ldots K$. This is without loss of generality since we could always increase the set of primary assets and then renumber them. Now, choose n > k and form the portfolio sequence.

$$W_i^n = \theta_{ii} (E_s(U_i) - R_F) / \sum_{j=K+1}^{n} (E_s(U_j) - R_F)^2 N_{jj}^2$$

for $i = k + 1, \ldots, \infty$

$$W_j^n = - \sum_{i=K+1}^{n} \lambda_{ij} W_i^n \text{ for } j = 1, \ldots, K$$

$$W_o^n = - \sum_{j=1}^{n} W_j^n + 1$$

By definition $\sum_{j=0}^{n} W_j^n = 1$ for all n.

Also $\sum_{j=0}^{n} W_j^n R_j = W_o^n R_F + \sum_{j=1}^{K} W_j^n R_j + \sum_{j=K+1}^{n} W_j^n R_j$

$$\because f_j = R_j \text{ for } j = 1, \ldots, K$$

$$U_j = R_j \text{ for } j = K+1, \ldots, n$$

$$\sum_{j=0}^{n} W_j^n R_j = - \sum_{j=1}^{n} W_j^n R_F + R_F + \sum_{j=1}^{K} W_j^n f_j + \sum_{j=K+1}^{n} W_j^n U_j$$

$$= R_F + \sum_{j=1}^{K} W_j^n (f_j - R_F) + \sum_{j=K+1}^{n} W_j^n (U_j - R_F)$$

$$= R_F - \sum_{j=1}^{K} \sum_{i=K+1}^{n} \lambda_{ij} W_i^n (f_j - R_F) + \sum_{j=K+1}^{n} W_j^n (U_j - R_F)$$

$$= R_F - \sum_{j=K+1}^{n} W_i^n \sum_{j=1}^{K} \lambda_{ij} (f_j - R_F) + \sum_{j=K+1}^{n} W_j^n (U_j - R_F)$$

$$\because R(X_i) = R_F + \sum_{j=1}^{K} \lambda_{ij} (f_j - R_F) + \theta_{ii} (U_i - R_F)$$

$$\therefore \sum_{i=K+1}^{n} R(X_i) W_i^n = \sum_{i=K+1}^{n} W_i^n R_F + \sum_{i=K+1}^{n} W_i^n \sum_{j=1}^{K} \lambda_{ij} (f_j - R_F) + \sum_{j=K+1}^{n} W_i^n \theta_{ii} (U_i - R_F)$$

$$\sum_{i=K+1}^{n} W_i{}^n \sum_{j=1}^{K} \lambda_{ij}(f_i - R_F) = \sum_{i=K+1}^{n} R(X_i) W_i{}^n - \sum_{i=K+1}^{n} W_i{}^n R_F$$

$$- \sum_{i=K+1}^{n} W_i{}^n \quad \theta_{ii} (U_i - R_F)$$

$$\sum_{j=0}^{n} W_i{}^n R_j = R_F - \sum_{i=K+1}^{n} R(X_i) W_i{}^n + \sum_{i=K+1}^{n} W_i{}^n R_F + \sum_{j=K+1}^{n} W_i{}^n \quad \theta_{ii}(U_i - R_F)$$

$$+ \sum_{j=K+1}^{n} W_i{}^n (U_j - R_F)$$

$$= R_F - \sum_{i=K+1}^{n} W_i{}^n \ (U_j - R_F) + \sum_{j=K+1}^{n} W_j{}^n (U_j - R_F) +$$

$$\sum_{i=K+1}^{n} W_j{}^n \quad \theta_{ii}(U_i - R_F)$$

$$= R_F + \sum_{i=K+1}^{n} W_j{}^n \ \theta_{ii}(U_i - R_F) \quad \text{for all n.}$$

$$\therefore \sum_{j=0}^{n} W_j{}^n \ E_s(R_j) = R_F + \sum_{j=K+1}^{n} W_i{}^n \quad \theta_{ii}(E_s(U_i) - R_F)$$

$$\because \quad W_i{}^n = \quad \theta_{ii}(E_s(U_i) - R_F) \ / \ \sum_{j=K+1}^{n} (E_s(U_j) - R_F)^2 N_{ij}{}^2 \ \forall i = K+1, \ldots, \infty$$

$$\sum_{j=0}^{n} W_j{}^n E_s(R(X_j)) = R_F + 1 \qquad \forall n$$

Also $Var_s(\sum_{j=0}^{n} W_j{}^n R(X_j)) = \sum_{i=K+1}^{n} (W_i{}^n)^2 \ Var_s(\ \theta_{ii}(U_i - R_F))$

by A.11 $Var_s(\quad \sum_{j=0}^{n} W_j{}^n R(X_j)) \ < \ \sigma^2 \quad \sum_{i=K+1}^{n} \ (W_i{}^n)^2$

$$= \sigma^2 \ / \ \sum_{i=K+1}^{n} (E_s(U_i) - R_F)^2 \ \theta_{ii}{}^2$$

$$= 0 \text{ as } n \to \infty$$

$$R_F + 1 > R_F$$

$\therefore$ Contradicts the no infinite asset arbitrage opportunity assumption

Since $\quad \lim_{n \to \infty} \sum_{j=1}^{n} \ \theta_j (E_s(U_j) - R_F)^2 < +\infty$

Most of the terms in the sum must be close to zero.

On the average, $\theta_j\,(E_s(U_j) - R_F) \doteqdot 0$

$$E_s(R(X_j)) - R_F \doteqdot \sum_{j-1}^{K} \lambda_j(E_s(f_K) - R_F) \qquad \forall\ s.$$

In other words, $E(f_K)) - R_F$ can be interpreted as the factor risk premium(8). This refers to the excess return the market has to give to the investor in order to compensate the investor for bearing this systematic risk, and λ_j is the coefficient called factor loading, which reflects the influence of factor j in the return of asset i. Since the primary assets can span M, if all investors believe the linear generation process, and when there is no arbitrage opportunity, then, to all investors, the expected return on any asset in M will be a linear combination of the factor loadings. This result will hold even in disequilibrium situation. However, if we add some equilibrium conditions on the above model, we can show that to all investors and for all assets, the excess return on any asset is a linear combination of the factor loadings (9).

i.e. $E(R(X_j)) \doteqdot R_F + \sum_{K-1}^{n} \lambda_{jk}(E(f_K) - R_F) \qquad \forall$ i and j.

Some of the assumptions above have to be modified and some new assumptions have to be added before we have this much stronger result. We call it the equilibrium version of APT (equi. APT). The assumptions which have to be modified are :

A.5 Given an event H, $Prob_i(H) = Prob_j(H)$ for all investors i and j, i.e. all investors agree on the expected value and covariance of the assets' cash flow.

A.11 We impose on e more restriction on the linear factor hypothesis. We require that the f_k, U_i, U_j are statistically independent for all i, j, k.

The new assumptions that have to included are :

A.12 All investors are risk averse and the investor's risk aversion is uniformly bounded by a constant ψ^i , which is independent of the particular economy n. Since the local measure of the ith investor's risk aversion is

$$-[E(\partial^2 V^i(C_1^i))/\partial C_1^2]/E(\partial^2 V^i(C_1^i)/\partial C_1)$$

therefore this assumption requires that

$$-\partial^2 V^i(C_1^i+\alpha)/\partial^2 C_1^2/E(\partial V^i(C_1^i)/\partial C_1) \leq \psi^i$$

A.13 We would like to use a sequence of market to approach the market economy. For example, M^{n+1} contains n+1 assets and the n+1th asset is not included in M^n. Also, we require that there exists a finite basis set of traded assets $\{U_i\}_{i=0}^{m} \in M^n$ where m<n.

Given the above assumptions, we can show that every investor holds a portfolio containing all the traded basis assets j=1, ..., m [10]. In other words, investors hold diversified portfolios. Before we prove the equi. APT, we would like to state that the residual excess expected return is nonzero and is bounded above by a constant[11]. i.e.

$$0<|\lambda_j, K+jE(U_j-R_F)|\leq\Phi(MKT_{j+k}(m))/|\lambda_j, K+j| \text{ for } j=1,\ldots,m-k$$

$$\text{where } MKT_{j+K}(m)=\sum_{j=1}^{n}\hat{N}^{i}_{j+K}P(Z_{j+K}/\sum_{j=j+K}^{m-K}(\sum_{i=1}^{n}\hat{N}^{I}_{j+K}P(Z_{j+K}))$$

and ϕ is a constant.

Therefore, if $\lim_{n\to\infty} MKT_j(m) = 0$ for $j=k+1,\ldots,\infty$

then, $\lim_{n\to\infty}|\lambda_{j,K+j}E(U_j-R_F)|=0$ for $j=1,\ldots,\infty$

This follow that $\lim_{n\to\infty}|E(R(X_j))-R_F-\sum_{K=1}^{K}\lambda_{jk}(E(f_K)-R_F)|=0\forall_j$

In other words, if the jth basis asset constitutes an insignificant part of the market portfolio $MKT_j(m)\approx 0$, then its idiosyncratic risk is diversifiable and it receives approximately zero excess expected return in equilibrium. As a result, for every primary asset j. $E(R(X_j))\approx R_F+\sum_{K=1}^{K}\lambda_{jk}(E(f_K)-R_F)$

However, this stronger result is only valid in equilibrium.

Some economists argue that the CAPM is just a special case of the APT. They treat the market portfolio as the only common factor in the APT and write the factor loading as the CAPM beta. i.e. $(E(R(X_j)) = R_F + \beta_j(E(R(m)) - R_F)$. Here E(R((m)) is the expected return on the market portfolio while βj is

$$cov(Rm, R(X_j)) / Var (Rm)$$

However, these two theories in fact are based on quite different assumptions. So we cannot draw the conclusion that CAPM is a special case of the APT. In fact, based only the assumptions of APT, one cannot derive the CAPM.

Apart from this, the APT still has several advantages over the CAPM. As we can see from above that the major conclusion of the APT holds not only in equilibrium condition but also in disequilibrium conditions. However, the CAPM is only and equilibrium model. Besides, the market portfolio does not play any particular role in the APT as it does in the CAPM. Furthermore, in the equi APT, all investors will hold a well diversified market portfolio. But them, the weights of different assets in this portfolio are not the same for different investors. These differences in portfolio composition persist due to differences in preferences. This contrasts with the CAPM, where in equilibrium, by the mean-variance assumption all investors hold the same combination of the market portfolio and the riskless asset[12].Since the CAPM and APT are derived from different assumptions, on theoretical ground, it is hard to justify which one is better. Therefore, the only way to compare them is by their explanatory power which forms one of our tests in the following section.

The testability of the APT is still very controversial among economists. In a strict sense, the weaker form of APT is not testable. Since it is based on the proposition that requires

$$\lim_{n\to\infty} \sum_{j-1}^{n} \theta_j (E_S(U_j) - R_F)^2 < +\infty$$

in testing the APT, we have to examine infinite assets. It is impossible in practice. However, if we use the equilibrium version of the APT even though the assumption is much stronger, the theory is potentially testable[13]. Since the equi. APT has a finite asset economy analogy, the upper bound of the excess return on the residual for finite assets is potentially testable. Evidence consistent with the this statement would support the finite asset equilibrium version of the APT. In fact, our empirical work on the APT will only focus on the equi. APT.

Chapter Two

Footnotes

(1) Ross, S.A. (1977) pp.189-190

(2) Copeland and Weston (1988) pp.198-199

(3) See Ross. S.A. (1977)

(4) See Jarrow, Robert A. (1988) Ch.8-9.

(5) Jarrow, Robert A. (1988) pp.94-100, pp.117-118

(6) Jarrow, Robert A. (1988) pp.104-108

(7) Jarrow. Robert A. (1988) pp.121-125

(8) See Admati and Pfleiderer (1985)

(9) See Jarrow, Robert A. (1988) Ch. 18

(10) Jarrow. Robert A. (1988) pp.270, pp.276-277

(11) Jarrow. Robert A. (1988) pp.271, pp.277-278

(12) Jarrow, Robert A. (1988) pp.270

(13) Jarrow, Robert A. (1988) pp.273-274

Chapter Three The Empirical Models

Roll and Ross(1980) is perhaps the first influential empirical study of the APT. They used the daily returns of 1260 securities (42 groups of 30 each) listed on the New York or American Exchanges from 1962 to 1972 in order to test the APT. They found at least three and probably four "priced" factors in the return generating process and the expected returns depend on the estimated factor loadings. They also compared the APT with the "own" variance effect and they concluded that "own" standard deviation does not add any further explanatory power to that of the factor loadings. Similar results were daily return data during the 1963 - 78 period, they found that not only the "own" variance effect but also the firm size effect do not contribute additional explanatory power to that of the factor loadings. He also found that the APT performs well in comparing to the CPAM. Bower, Bower and Logue(1984) by using the monthly data of all stocks continuously traded on the New York and American Stock Exchanges during the 1971-1979 periods, they also found that the APT is better than the CAPM in explaining and forecasting return variations through time and across assets . However, the work conducted by Lehmann and Modest(1988) was having result which is quite different from the others. They found that the APT cannot account for the firm size effect. Nevertheless, they still found that the APT provides an adequate account for the "own" variance and the dividend-yield effects where the CAPM fails. Therefore, they concluded that the APT is pricing most listed equities with little errors in comparing with the CAPM.

To my knowledge, K.N.Lee(1989) is the first work in adapting the APT to measure the systematic risk associated with the Hotel Industry in Hong Kong. He explicitly specified that there are four common factors exist in the Hotel sector. They are the industrial production rate of U.S. and Japan, the exchange rate, the concentration ratio of hotel industry and the political environment. However, He assumed that the market structure is constant and the political environment is stable over the sample period. As a result he only took into account the CAPM market index, the industrial production and the exchange rate into this model. Because of the problem of multicollinearity, he combined the industrial production and the exchange rate into an industry index. He then regressed the Hotel index on the market index as well as the industry index. From his work, he concluded that there exist other explanatory variables besides the CAPM market index in explaining the return on hotel stock. Therefore, the linear factor model, it is to say the APT, is better than the CAPM in measuring the systematic risk on hotel stock. However in interpreting his findings, we have to bear in mind that he was implicitly assuming that the idiosyncratic risks obey the Ross separating distribution (See WEI(1988)). Since it is quite difficult to test whether the above assumption is fulfilled or not, it is better to use other convenience method such as the one suggested by CHEN(1983) to compare the performance of the APT and the CAPM. Besides, although it is worthwhile for us to identify the common factors, the more fundamental questions are whether the APT is suitable in applying to our stock market and what is the number of the common factors. Our empirical models are devoted to answering these basic questions.

Although there are a total 33 constituent stocks in the Hang Seng Index, we only select 32 stocks in our analysis since

TVB have missing values during our sample period. We collect the daily closing prices of these stocks listed on the Hong Kong Stock Exchange from 1/12/1987 to 31/3/1990. Since all stock prices are adjusted for dividends and stock splits.[(1)], therefore we calculate the daily returns of these stocks simply as their capital gains, i.e. $R_t = (Pt - P_t-1)/P_t-1$, where R_t is the return on date t, P_t-1 and P_t are the prices of this asset on date t-1 and date t respectively. We have to notice that in using the daily closing prices, we lose the information about the price fluctuation within a date. It is better to use the average of the high and the low price of the date instead of the closing price. However, due to the budget constraint of this study, we choose the closing price. In fact, a lot of the empirical work concerning the Hong Kong stock market also used the daily closing prices(see Law(1982), Wong and Kwong(1984), P.M. Chan (1990)). In obtaining the CAPM Beta for different stocks, we adopt the Hang Seng Index as proxy for the market index. We choose the Hang Seng index since it has consistently represented about 75% to 80% of the market in terms of market value as well as turnover and it is widely accepted. Therefore, it could fairly be regarded as representative of the market[2].

The equilibrium version of the APT and CAPM are the basic models that will be tested using data on equity daily rates of return for 32 listed stocks in the Hong Kong Stock Exchange. The APT states that if asset returns follow a linear generating process, then the expected returns of these assets will be approximately linear to the factors loadings. In other words,

If $$R_i = E_i + \sum_{j=1}^{K} b_{ij}\sigma_j + \varepsilon_i \qquad (1)$$ then

$$E_i \approx \lambda_0 + \sum_{i=1}^{K} \lambda_i b_{ij} \qquad (2)$$

where

(i) R_i : the random rate of return on asset i.

(ii) E_i : the expected rate of return on asset i.

(iii) b_{ij} : the factor loading of the common factor j to the asset i. This is used to measure the sensitivity of that asset to the change of the common factor j.

(iv) σ_i : normalized common factor j with variance equal to 1. i.e. $E(\sigma_i) = 0$ and $Var(\sigma_i) = 1$

(v) ε_i : this is used to measure the unsystematic risk which is idiosyncratic to the ith asset. It is supposed to be a noise term and can be diversified.

i.e. $E(\varepsilon_i)=0, E(\varepsilon_i\varepsilon_j)=0 \forall i\neq j$

$$E(\varepsilon_i\varepsilon_j)=\sigma^2\leq\bar{\sigma}^2, \forall i=j$$

(vi) λ_j : As we stated in Ch.2, λ_j can be interpreted as the risk premium of the common factor j. This parameter reflects the excess return on the jth common factor.

i.e. $\lambda_j = E(f_j) - R_F$, where R_F is the risk free rate of return and λo is corresponding to R_F.

We rewrite equation (2) as an exact equality.

i.e. $$E_i=R_F+\sum_{j=1}^{K}\lambda_{ij}b_{ij} \qquad (3)$$

Equation 3 forms the basis of our empirical test on the APT.

On the other hand the CAPM states that the expected return is linearly related to the return on the market portfolio.

i.e. $$E_i=\{R_F+\lambda Cov(R_i,R_m)\}/\sigma(R_m) \qquad (4)$$

where (i) E_i is the expected return on the ith asset.

(ii) R_F is the risk-free rate of return.

(iii) λ is the market risk premium per unit of risk.

i.e. $\lambda = (E(R(m)) - R_F) / \sigma R(m)$

(iv) R_j is the random rate of return on asset i.

(v) R(m) is the random rate of return on the market portfolio.

Equation 4 forms the basis for our empirical test on the CAPM.

In deriving the CAPM as well as the equilibrium APT, we assume that investors have homogeneous belief. Therefore, investors agree on the rate of return, the factor coefficient, the expected return together with the variance and covariance of the return. As a result, the pricing equation which holds for the individual will hold at the market level as well. Besides, this result permits us to use the ex post data to test these ex ante theories.

The stages involved in testing the APT are outlined as follows [3] :

(1) For a group of assets; (in this case, a group of 32 selected stocks), we compute the sample covariance matrix from the time series of return.

(2) We apply the maximum likelihood factor analysis on this covariance matrix in order to estimate the factor loading b_{ij} as well as the number of common factors.

(3) The individual asset factor loading estimates from the previous step are used to explain the cross-sectional variation of individual estimated expected returns.

(4) Estimates from the previous step are used to measure the statistical significance of risk premia associated with the estimated factors.

We prefer the maximum likelihood factor analysis method since (i) more is known about its statistical properties and (ii) this method provides the capability of estimating the number of latented common factors [4].

The first hypothesis to be tested is the pricing equation (3)

Ho : $\lambda_1 = \lambda_2 = \ldots\ldots = \lambda_K = 0$

H1 : at least one of them is not equal to zero

The factor model of APT can be written in matrix form, as

$R_t = E+B\sigma_t+E_t$ and the arbitrage pricing theory requires $E=\lambda_0+B\lambda$

Therefore, the regression model is

$$\bar{R}_t=\lambda_0+\hat{B}\lambda+U_t \tag{5}$$

where U_t is the disturbance at date t and B is obtained from step 2.

It might seen natural to test this hypothesis by first using the arithmetic mean return of R_t, say $\bar{R}_t$ to estimate the expected return E and then regress the estimated expected return on the factor loading by using a simple OLS cross - sectional regression. However, this procedure might produce result that is biased towards the alternative hypothesis. It is to say, we might find risk premia for "priced" factors,even when their true prices are actually zero. Since the average value of σ_t , say $\bar{\sigma}=\sum_t \sigma_t/T$ is not be exactly zero in any sample. Therefore, the cross sectional regression (5) should be written as

$$\bar{R}=\hat{B}(\lambda+\bar{\sigma})+\bar{\hat{E}}$$

$$\Leftrightarrow \quad \bar{R}=\hat{B}\hat{\lambda}+\bar{\hat{E}}$$

so that $E(\hat{\lambda}) = \lambda + \bar{\sigma}$. It follows that so long as $\bar{\sigma}$ is not exactly zero, $E(\hat{\lambda}) \neq 0$ even when $\lambda = 0$

To correct this problem, we have employed a method analogous to that of Roll and Ross (1980).

Since if the APT is true, $E=\lambda_0+B\lambda$, therefore

$$R_t=\lambda_0+B\lambda+B\sigma_t+E_t$$

$$\Leftrightarrow \quad R_t=\lambda_0+B\lambda+(B\sigma_t+E_t)$$

$$\Leftrightarrow \quad R_t=\lambda_0+B\lambda+U_t \qquad (6)$$

We estimate a cross-sectional regression like (6) for every time period t. $R_t=\hat{B}\hat{\lambda}_t+\hat{E}_t$ $\hat{\lambda}_t$

and then use the time series of $\hat{\lambda}_t$ to estimate the standard error of the average value of λ. This yields an inference about whether the true λ is non-zero[5].

The second test performed on the APT is the test of the APT against a specific alternative[6]. If the APT is correct, the cross-sectional variation in the expected return should be fully explained by the common factors. Therefore, if we find a variable which still has significant influence on the expected return even when the effects of the common factors are accounted for, then we can reject the null hypothesis that APT is true. This test is stronger than the first test since it involves an alternative hypothesis stating that other factors have significant effects on the expected return. In this test we choose one particular variable, the variance of individual returns, or the "own" variance.

The own variance is chosen because of the well-known correlation between average returns and own variance. The total variance would not affect expected return if the APT is valid. It is simply because its diversifiable component will not be priced in equilibrium and its non-diversifiable part would be captured by the common factors. However, a possible source of the effect of the own variance on expected return is skewness in the distribution of individual returns. Positive skewness can create positive dependence between the sample mean and sample variance and conversely for negative skewness. Therefore, we have to minimize the effect from skewness when we preform our second test on the APT. A procedure which can be used to reduce the effect of skewness is to estimate each parameter from a different set of obser vations. We use observations 1, 7, 13, 19 ... to estimate the expected return, observations 3, 9, 15, 21 ... to estimate the factor loadings, and 5, 11, 17, 23, ... to estimate the own variance, $S_j = \sum_{t-1}^{T} [(R_{jt}-R_j)^2/T]$, After correcting the skewness in the distribution of individual return, we can run a simple O.L.S. cross-sectional regression on the following model :

$$R_j-\lambda_0+\lambda_1 b_{1j}+\ldots+\lambda_K b_{kj}+\lambda_s S_j+U_j,$$

$j= 1, \ldots, 32$

where R_j is the sample arithmetic mean return for security j, b_{kj} is security j's loading on factor k, the λ's are the regression coefficients. S_j is individual asset j's total standard deviation of daily return during the sample period and U_j is a residual. This can yield an influence whether the "own" variance have any significant effects in determining the expected return. However, due to the same deficiency stated in the former test, we apply the same procedure in test 2 as we did in test 1. We use observations 1, 7, 13, 19, ... to perform the cross-sectional regression on the following model :

$$R_{jt} = \hat{\lambda}_0 t + \hat{\lambda}_{1t} b_{1j} + \ldots + \hat{\lambda}_{Kt} b_{Kj} + \hat{\lambda}_{st} S_j + U_{jt},$$

j = 1, ... , 32

t = 1, 7, 13, 19 ...

This yields time series data of $\hat{\lambda}_{st}$, which represent the effect of "own" variance. The time series of $\hat{\lambda}_{st}$ is used to compute a standard error for the mean value, i.e. for $\bar{\hat{\lambda}} = \sum_t \hat{\lambda}_{st}/T$ in order to test for the significance of an "own" variance effect.

The third test we will perform concerns the question on which of the models, CAPM or APT, is specified more correctly ? In answering this, we first perform a covariance analysis on the return matrix in order to find the COV(Rj , RHSI) and Var(RHSI). Rj is the return on security j and RHSI is the return of the Hang Seng Index which is used as a proxy for the market portfolio. Since $B_j = Cov(R_j, RHSI)/Var(RHSI)$,

therefore we can find the CAPM Beta for different asset j.

Secondly, we calculate the mean returns of every two even day returns, Rj, and then running a cross-sectional regression

$\bar{R}_j = r_0 + r_1 B_j + e_j,$

j=1, ... , 32.

Therefore , we can generate a time series of the estimated expected returns which is generated by the CAPM,

($\hat{r}_j$, CAPM, j = 1, ... , 32)

Thirdly, we use the even day returns to estimate the factor loadings, and then we regress the Rj obtained in the above stage on the factor loadings. Therefore we obtain a time series of the estimated expected return of the APT i.e.($\hat{r}_j$ APT. j= 1, ... .32)

Finally, since the APT and the CAPM are two unnested linear models. therefore, we can use the J test suggested by Davidson and Mackinnon (7). The J test procedure requires us to regress the mean return on the factor loadings of APT and the predicted value of expected return generated by the CAPM. That is to say, we perform the following regression model :

$$\hat{R}_j = (1-\alpha)(\lambda_0 + \lambda_2 b_{2j}) + \alpha \hat{r}_{j,\ capm}$$

If the APT is specified correctly relative to the CAPM, then the true value of α is equal to zero. We estimate the mean and standard error of $\hat{\alpha}$ directly from its time series. We can obtain this by performing the above regression in sub-intervals. Each interval contains five (even) days. By the mean and the standard error of $\hat{\alpha}$. We can simply test whether $\alpha=0$ by using the t-test. We have to note that the t statistics form the above regression is conditional on the truth of the APT, not on the truth of the CAPM. Therefore, for completeness, we reverse the role of the APT and the CAPM in the above regression. It is to say, we regress the mean return on the CAPM Beta and predicted value generated by the APT. When this is done, it is conceivable that both hypotheses may be rejected, or that neither may be rejected, or that one may be rejected and the other may not be. However, the J-test procedure is really designed for testing model specification, not for choosing among the competing models. Therefore, in order to compare the ability in explaining data of the APT and the CAPM. We have to perform a direct test between them and that is the central idea of Test Four.

The final test we will perform is a direct test of APT against CAPM[8]. If the CAPM is not misspecified, the expected return of asset i would be captured by Bi and the residual would behave as the white noise. If the CAPM is misspecified and the Bi does not capture all the information about expected return. The remaining part will be contained in the residual. If there is another model which can capture this part of information, then the residual will be priced by that model. In order to compare the performance of CAPM and APT, we run a regression with the CAPM residual as the dependent variable and the factor loadings of APT as the independent variables. For completeness, we also regress the residual from APT on the CAPM Betas. Both of the residuals are calculated from the even day returns. For the CAPM to be better, the beta must be able to capture all the priced information and render all the factor loadings insignificant in explaining the CAPM residual. On the contrary, if the APT is better in capturing the price information, then the CAPM betas should not have any explanatory power in explaining the APT residual.

Although the above tests involve the comparison between the CAPM and the APT, the acceptance of APT does not mean that we reject the CAPM. They are dependent on different assumptions and these assumptions are not mutually exclusive to each other. They are just different measurements of the systematic risk. We accept the APT simply because it is better in explaining the data. In other words, in explaining the cross-sectional variation in expected return, the APT is better than the CAPM.

Chapter Three

Footnotes

(1)The adjusted data are supplied by Compuserve Consultants Ltd.

(2)Chan P.M. (1990) pp.8

(3)Roll and Ross (1980) PP.1086-1087

(4)Roll and Ross (1980) PP.1087

(5) Roll and Ross (1980) PP.1089-1093

(6)Roll and Ross (1980) PP.1093-1098

(7)Davidson and Mackinnon (1981) PP. 781-783

(8)Chen, N.F. (1983) PP. 1402-1405

Chapter Four Empirical Results

In test 1, we use the maximum likelihood method to determine the number of factors and the factor loadings. We include more factors than necessary, since we can use the regression method to determine the right number of factors. The reluctant factors will not be priced under the regression analysis. However, if we retain too few factors during the maximum likelihood stage, we have to bear the risk that some priced factors are excluded from our analysis. Under the null hypothesis that 5 factors are sufficient to represent the data set, we obtain a chi-square statistic equal to 561 and the corresponding degree of freedom is 346. Therefore, five factors are retained from the maximum likelihood stage. We have to note that five factors is the maximum number of factors we can have, since for a number of factors greater than 5, the communality is greater than 1. It follows that maximum likelihood method will break down for a number of factors larger than 5. In completing our test, we use the daily return as the dependent variable and use the factor loadings as the independent variables. We then perform a cross-sectional regression for each day. From the time series of the estimated coefficient, only the coefficient corresponding to Factor Two is significantly different from zero at 2% level of significance. Other coefficients are not significantly different from zero even at 50% level of significance. The statistics are summarized in table 1. However, since the alternative hypothesis is just that at least one risk premium is not equal to zero, therefore, we can confidently reject the null hypothesis that all the risk premia are equal to zero and accept the alternative hypothesis. However, our result is quite different from the previous work (Roll and Ross). They usually find three to five factors which are priced significantly different from zero. Nevertheless, our result still confirm to the APT.

In conducting test 2, we first notice that the mean even day return is highly correlated with the variance. We perform a simple OLS between the variance and the mean return. We find that the variance effect is very significant(see table 2).We also perform a simple OLS between the variance and the skewness and find that they are highly correlated (see table 2). However, we find that the correlation between skewness and mean return is not so obvious (see table 2). Therefore, we suspect that the skewness does not create a positive or negative correlation between the mean return and variance. In other words, we may not need to correct for the skewness in our model. Therefore, it is possible to conduct test 2 simply by regressing the mean return directly on the factor loading, in our case factor 2, and on the variance.

We apply maximum likelihood method on observations 3, 9, 15, 21, ... , and four factors have been retained. We preserve the second , third and the fourth factor after preforming the regression analysis. The factor loadings and the t-value of the coefficient associate with the factor loadings are reported in table 3. However, we find that the t-value is smaller in comparing with the result in test 1. Although Factor 2 is still the most significant factor, the coefficient corresponding to it is only roughly significant at 30% level of significance. We use these factor loadings and the variance calculated from observation 5, 11, 17, 23 ... as independent variables and use the mean return on 1, 7, 13, 19, ... as dependent variable. We perform the regression analysis and find that the own variance effect is very insignificant (see table 4). We also directly regress the return on 1,7,13,19 ...... on the factor loadings and their own variances. From the time series of the estimated parameter corresponding to the own variance, we found that the own variance effect is also very insignificant(see table 5A). We also found that the estimators associated with the factor loading are quite robust since even if we drop the own variance in our empirical model, their values still remain roughly the same (see table 5B). Although the factor loadings in explaining the variation in return is better than the own variance, in the above two tests their effects are only significantly different from zero at 30% level of significance. If we do not correct for the skewness, we find that the factor loading is very significant in explaining the variation in the cross-sectional mean returns while the effect of the own variance, on the contrary, is very insignificant. The resulting statistics are in table 6 . As a result, we conclude that the APT is correct in the presence of other hypothesis. In other words, the APT can account for the own variance effect.

In test three, we apply the maximum likelihood method on the even day returns and four factors have been retained. After we regress the even day returns on the factor loadings, the first and the second factors are preserved. Factor two is still the most significant factor (see table 7B). The predicted return generated from the APT, $\hat{r}$APT is calculated from the above procedure. We also calculated the CAPM Beta which is reported in table 6B. As in other previous work, the real estate sector usually has a Beta greater than 1 and the Banking sector have a Beta less than 1 (see table 8C). We also use these Beta to generate the predicted return of the CAPM, $\hat{r}$CAPM. From the results, both the $\hat{\alpha}$ of the $\hat{r}$APT and that of $\hat{r}$CAPM are all significantly different from zero. Therefore, the J test tells us that both the APT and the CAPM have model specification errors relative to each other. As a result, we cannot judge whether the APT is better specified in the presence of the CAPM or not from the model specification criterion. The resulting statistics are summarized in Table 9. Since both the APT and the CAPM explicitly or implicitly assume that returns are generated by a linear process (see Chen (1983)), they all postulate that expected returns are linearly related to the CAPM Beta or the factor loadings.

However, the return generation process may not be the one as we assumed. If this is true, the relationship between expected return and the independent variables may not be linear or some important variables may be omitted. Therefore, it seems likely that they both have their own model specification errors.

Finally, in test 4, we find that the factor loadings can significantly explain the residual of the CAPM, say RCAPM. However, the CAPM Beta do not have any explanation power in explaining the residual of the APT, i.e. RAPT. Both the RCAPM and RAPT are obtained in test 3 and the result of this test is reported in table 10. As a result, we can conclude that the APT can account for some systematic risk which cannot be captured by the CAPM. Therefore the APT is better than the CAPM in measuring the systematic risk.

We can summarize the empirical findings in four points. Firstly, there exists definitely one common factor in the return generation process. No matter which subset of our data set we choose, we always find that factor 2 is significant in explaining the variation in the expected returns of different assets. Although we only find one common factor, we should not misinterpret it as the CAPM market factor. They are in fact two distinct concepts. Secondly, after we account for the effect of the common factor, we find that the own variance effect does not contribute any additional explanatory power to that of the factor loadings. Thirdly, we cannot find evidence to support that the model specification of the APT is better than the CAPM. It seems that they all have model specification errors in the presence of the other. Lastly, the APT has better performance in measuring the systematic risk than the CAPM. We find that the factor loadings can account for more systematic risk than the CAPM.

In interpreting our empirical results, we have to pay attention on the following two points. Firstly, some of the assumptions behind our empirical model are not fulfilled in the actual world. The proportion of institutional investors still remain low in our market and individual investors dominate the stock market(1). Therefore, the transaction costs associated with the trading may be quite significant to these individual traders. Besides , short selling of stocks is still illegal in Hong Kong. Although investors can make use of the index future market to hedge against risks, it is not a convenient and efficient way to do so. Under the current regulations, investors who hold stocks can sell index futures contracts to hedge. But investors who have bought futures contracts cannot short sell stocks for the same purpose[2]. Because of the restriction in short selling and the transaction cost involved in trading stocks, individual investors may find it quite difficult to take arbitrage positions. Therefore, our stock market is not as efficient as the stock market in United States. Some empirical findings in Hong Kong even conclude that weak-form market efficiency does not hold in our market. (see Law(1982), Wong and Kwong(1984)). In the other words, if return is higher than the expected level, it will take some time to drive out the abnormal profit. As a result, arbitrage opportunities may exist quite often in our market.

Apart form this, since it is difficult for individual traders to form well diversified portfolios. We suspect that not all of the unsystematic risks can be diversified away. Therefore,not only the systematic risk, but also the unsystematic have influence in determining the expected return. In addition to this, since our market is rather small relative to that of United State, Japan, United Kingdom in terms of capitalization and family control is quite common here, most securities are not held and traded by many investors(3). In other words, not all stocks in Hong Kong are marketable. Since a lot of the assumptions are violated in the real world, it follows that the empirical relationship between risk and returns suggested by the APT or the CAPM may not hold as they should in theory. Although the assumptions imposed are an idealization, it is introduced to simplify the analysis. Besides, an ideal model provides us a benchmark against which actual market can be compared. Unless we understand how the idealized market works, we cannot possibly understand how the more complex market works(4). After all, we have to remember that we should base our test on the conclusions of the theory rather than the assumptions in deriving them.

The second point we have to consider is the special feature of our market. Our economy is heavily dependent on international trade, the value of the total real exports of goods and services amount to 363.8 billion Hong Kong dollars whereas the real GDP in 1988 was about 246 billion Hong Kong dollars. Since our major trade partners are United State, China, Japan and U.K., economic conditions in these countries will definitely affect our economic performance. It follows that we may make use of the industrial production rate in these countries and the exchange rate to measure the common factor in the return generation process. Besides, the history of our stock market tells us that it is easily affected by political events in our neighboring countries, especially mainland China(5). Political events occurring in the North affect the stability of our community and influence our performance. If we do not have the confidence in the stability of Hong Kong, then it is natural that the value of all assets located in here will decline. The reason is simply that the expected income stream of these assets will be lower and investors do not have the incentive to hold them. As political risks affect the whole stock market, therefore, we can classify it as systematic risk. During our sample period, the stock market performance was heavily influenced by political uncertainty of Hong Kong. The most significant example is the June 4 event. Therefore, the political risk may be quite significant in this sample period. As a result, although we do not intend to identify the common factor in our work, we suspect that the common factor we found may represent the combination effects of the industrial production rate, exchange rate as well as the political risk.

Lastly, since the whole sample period is within a bear time. The Hang Seng Index in 1/12/87 was 2108.55 and it was only up to 2997.98 in 31/3/90. The variation of the stocks' return was not so obvious after crash. If we can perform our test in a longer time series and can cover more stocks, then we may have much better results.

TABLE 1

Sumarized Statistics of Test One

Rjt = Xo + X1b1j + X2b2j + X3b3j +X4b4j + X5b5j + Uj,

j = 1, 2, ... , 32

t = 1, 2, ... , 577

estimated coefficient	t-value	degree of freedom
X1 = 0.0002	0.12	575
X2 = 0.0031	2.36	575
X3 = 0.0008	0.48	575
X4 = 0.0005	0.33	575
X5 = 0.0003	0.24	575

TABLE 2

The effect of variance and skewness in explaining the return

(A) $Rj = a + bVarj + Uj$		
estimator b	t-value	degree of freedom
0.7702	2.098 (2.042)	31
(B) $Rj = a + bSkewj + Uj$		
estimator b	t-value	degree of freedom
- 0.0010	-0.915 (-2.042)	31
(C) $Varj = a + bSkewj + Uj$		
estimator b	t-value	degree of freedom
-0.0001	-2.795 (-2.042)	31

A, B and C are the regression model and the number within the bracket is the critical t-value at 5% level of significance with degree of freedom equal to 31.

TABLE 3A

Summarized statistics of the Maximum Likelihood stage of Test 2

Chi-square = 384149 degree of freedom = 374 these value are under the null hypothesis that 4 factors are sufficient in representing the data set

TABLE 3B

The simple test on the APT using observations 3,9,15,21...

Rjt = Xo + X1b1j + X2b2j + X3b3j + X4b4j + Uj

j = 1, 2, ... , 32

estimated coefficient	t-value	degree of freedom
X1 = -0.0026	-0.35	93
X2 = 0.0017	0.97	93
X3 = -0.0013	-0.80	93
X4 = 0.0009	0.73	93

Table 4

The result of test two with correction of skewness and using the mean return as dependent variable.

$Rj = Xo + X2b2j + X3b3j + X4b4j + XsVarij + Uj$

$j = 1, 2, 3, \ldots\ldots, 32$

estimated coefficent	t-value	degree of freedom
X2 = 0.0021	1.131	28
X3 = -0.0015	-0.702	28
X4 = 0.0010	0.585	28
Xs = -0.5490	-0.374	28

TABLE 5A

The results of test two with correction of skewness

$$Rjt = X0 + X2b2j + X3b3j + X4b4j + XsVari + Ujt$$

$j = 1, 2, \ldots, 32$

$t = 1, 7, 13, 19 \ldots$

estimated coefficient	t-value	degree of freedom
X2 = 0.0021	1.09	93
X3 = -0.0015	-0.97	93
X4 = 0.0097	0.80	93
X5 = -0.5486	-0.14	93

Table 5B

$Rjt = Xo + X2b2j + X3b3j + X4b4j + Ujt$ $j = 1, 2, 3, \ldots\ldots, 32$ $t = 1, 7, 13, 19 \ldots\ldots$		
estimated coeefficient	t-value	degree of freedom
X2 = 0.0020	1.34	94
X3 = -0.0017	-0.81	94
X4 = 0.0010	0.72	94

Table 6

The result of test 2 without correction of skewness

Rj = ao + a1b2j + a2 Varj + Uj, Varj is the variance of asset j.		
estimated coefficient	t-value	degree of freedom
a1=0.0030	4.496 (2.045)	29
a2=0.4410	1.490	29

The value inside the bracket is the critical t-value at 5% level of significance

TABLE 7A

Result of the Maximum Likelihood stage in Test 3

Ho : 4 factors are sufficient in representing the data set
Chi-square = 487.778
Degree of freedom = 374

TABLE 7B

The test on the APT using observation 2,4,6,...,576

$$Rjt = Xo + X1b1j + X2b2j + X3b3j + X4b4j + Uj,$$

$$j = 1, 2, \dots, 32$$

estimated coeficients	t-value	degree of freedom
X1 = 0.0019	0.82 a(0.842)	284
X2 = 0.0024	1.60 b(0.645)	284
X3 = 0.0006	0.31	284
X4 = 0.0010	0.58	284

a) critical t-value with level of significance = 40%
b) critical t-value with level of significance = 10%

TABLE 8A

CAPM BETAS CALCULATED FROM EVERY DAY RETURNS

Bi = Cov(Ri, Rm)/Var(Rm)

i = 1, 2, ... , 32

Rm = Return of the Hang Seng Index

B1	0.860682	B12	1.136101	B23	0.722973
B2	0.757400	B13	0.722973	B24	1.652510
B3	1.170528	B14	0.585264	B25	0.826255
B4	1.273810	B15	0.860682	B26	1.136101
B5	0.757400	B16	1.067246	B27	1.273810
B6	0.895110	B17	0.757400	B28	1.377092
B7	1.686937	B18	1.377092	B29	1.101637
B8	1.067246	B19	1.283810	B30	1.136101
B9	1.445946	B20	1.308237	B31	0.929537
B10	0.722973	B21	1.204955	B32	1.377092
B11	1.445946	B22	1.273810		

TABLE 8B

CAPM BETA CALCULATED FROM THE EVEN DAY RETURNS

B1	0.735478	B12	1.225797	B23	0.735478
B2	0.735478	B13	0.735478	B24	1.618052
B3	1.176765	B14	0.637414	B25	0.882574
B4	1.323861	B15	0.735478	B26	1.029670
B5	0.686446	B16	1.078701	B27	1.323861
B6	0.833542	B17	0.833542	B28	1.372893
B7	1.667084	B18	1.225797	B29	1.029669
B8	0.980638	B19	1.274829	B30	1.225797
B9	1.372893	B20	1.274839	B31	0.833542
B10	0.784510	B21	1.176765	B32	1.372893
B11	1.372893	B22	1.372893		

Table 8C

The list of the common stocks under consideration

B1 : Bank of East Asia	B17: HK Telcom
B2 : Cathay Pacific	B18: Hopewell
B3 : Carendish Int'l	B19: Hutchison Whampoa
B4 : Cheung Kong	B20: Hysan Development
B5 : China Light	B21: Jardine Matheson
B6 : Dairy Farm Int'l	B22: Jardine Strategic
B7 : Great Eagle Hldg	B23: Kowloon Motor Bus
B8 : HongKong Aircraft	B24: Lai Sun Int'l
B9 : Hang Lung Dev.	B25: Mandarin Oriental
B10: Hang Seng Bank	B26: Miramar Hotel
B11: Henderson Land land	B27: New World Dev Dev
B12: Hong Kong Hotels	B28: Sun Hung Kai Prop
B13: Hong Kong Bank	B29: Swire pacific 'A'
B14: Hong Kong Electric	B30: Wharf (Holdings)
B15: HK and China Gas	B31: Winsor Industrial
B16: Hong Kong Land Hldg.	B32: World International

Table 9

Resulting Statistics of Test Three

A)	$R_j = (1 - \alpha)(X_0 + X_1 b_{1j}) + \alpha \hat{r}j\text{CAPM}$	
estimated coefficient	t-value	Degree of freedom
$\alpha = 1.0004$	722.46 (2.660)	54
B)	$\tilde{R}_j = (1-\alpha)(a_0+a_1B)+\alpha\hat{r}_{j_{APT}}$	
estimated coefficient	t-value	Degree of freedom
$\alpha = 0.5320$	13.06 (2.660)	54

The value inside the bracket is the critical t-value with level of significance equal to 1%

TABLE 10

Summarized statistics of Test 4

(A) RCAPM = ao + a1b1 + a2b2 + U	
estimated parameters	t-value
a1 = -0.0005	-0.43
a2 = 0.0015	1.7 (1.64)

the number within the bracket is the critical t-value at 10% level of significant and degree of freedom = infinitive.

(B) RAPT = bo + b1B + U	
estimated parameters	t-value
b1 = -0.0002	-0.29

Chapter Four

Footnotes

(1) Securities Journal Feb. 1990 pp.18

(2) Ibid.

(3) Chan P.M. (1990) pp.14

(4) Jarrow R.A. (1988) pp.19

(5) Wong K.A. (1988) pp.57-79

CHAPTER Five CONCLUSION

The main implication of the APT and the CAPM is that, under efficient market conditions, investor should not bear unsystematic risk since this can be diversified. It is only the systematic risk which will be priced in the capital market. Both the CAPM and the APT offer a method in measuring the systematic risk. The CAPM uses the Beta value to reflect the influence of the systematic risk on the expected return while the APT uses the factor loadings to capture this effect. The CAPM states that in equilibrium, $E(R_j) = aR_f + bB_j$, on the other hand, the APT points out that $E(R_j) = X_0 + \sum_{i-1}^{K} X_i b_{ij}$ in equilibrium. It is theoretically hard to determine which model is better since they are derived from quite different assumptions. In fact, we can classify the CAPM as a mean-variance efficient model .The main results of APT on the other hand, do not depends on the mean-variance assumption at all. The major assumption of the APT is that no arbitrage opportunities exist. In term of testability, the CAPM is not testable and the equilibrium version APT is potentially testable.

By using 32 Hong Kong stocks data , the empirical results support the APT against both an unspecified alternative hypothesis (test 1) and the specific alternative hypothesis (test 2) that own variance has an independent explanatory effect on expected return. Therefore , based on the empirical evidence gathered so far, the APT cannot be rejected in favor of any alternative hypothesis. Test 3 reveals that neither the APT nor the CAPM is better in model specification in the presence of the other. From the result of test 4, the APT performs very well against the CAPM as implemented by the Hang Seng Index. The APT can capture more systematic risk and therefore it is better than the CAPM in measuring the systematic risk. Therefore, the APT is a reasonable model for explaining cross-sectional variation in asset returns.

Lastly, of course, effort should be directed towards identifying a more meaningful set of sufficient statistics for the underlying factors (see Chen, Roll and Ross(1986)). While this is not a necessary component of tests of the APT and therefore we do not include it in our analysis, it is an interesting and worthwhile pursuit of its own.

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